Can one **identify** two unital JB*-algebras by the **metric spaces** determined by their **unitary sets**?

A Hatori-Molnár type theorem for unital JB*-algebras

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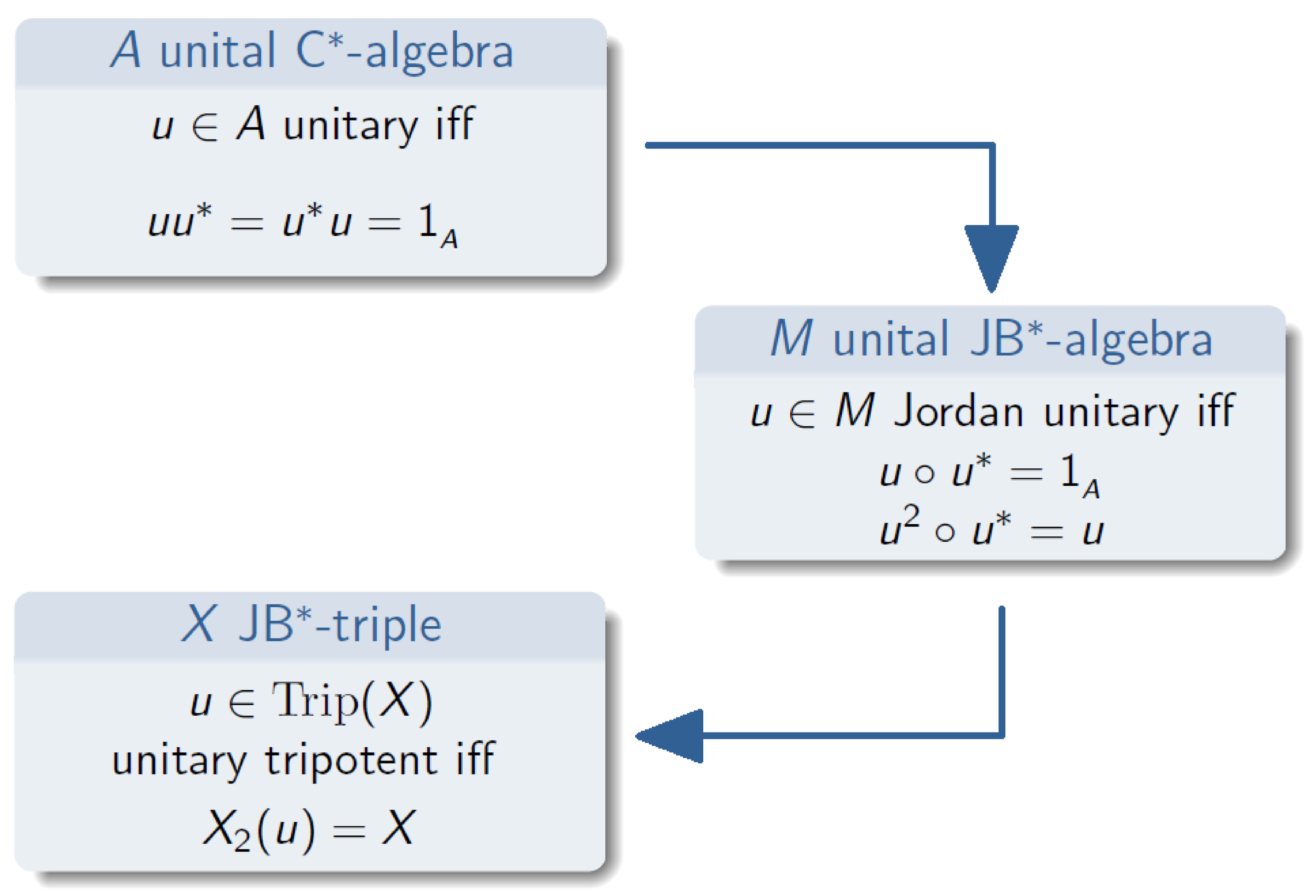
[JB*-algebras]

M complex Jordan Banach algebra
$^* : M \to M$ involution
$\|U_a(a^*)\| = \|a\|^3, \quad \forall a \in M$

where $U_a(x) = 2(a \circ x) \circ a - a^2 \circ x, \quad (x \in M)$



Theorem

Let M and N be two unital JB*-algebras, and let $\mathcal{U}(M)$ and $\mathcal{U}(N)$ denote the sets of all unitaries in M and N, respectively. Suppose $\Delta : \mathcal{U}(M) \to \mathcal{U}(N)$ is a surjective isometry, and that one of the following holds:

- $\|\mathbf{1}_N - \Delta(\mathbf{1}_M)\| < 2$;
- $\exists\, \omega_0 \in \mathcal{U}(M)$ such that $U_{\omega_0}(\Delta(\mathbf{1}_M)) = \mathbf{1}_N$.

Then there exists a unitary ω in N satisfying

$$\Delta(e^{iM_{sa}}) = U_{\omega^*}(e^{iN_{sa}}).$$

Furthermore, there exist a Jordan *-isomorphism $\Phi : M \to N$, and a central projection p in N such that

$$\Delta(e^{ih}) = U_{\omega^*}\left(p \circ \Phi(e^{ih})\right) + U_{\omega^*}\left((\mathbf{1}_N - p) \circ \Phi(e^{ih})^*\right), \quad \forall h \in M_{sa}.$$

Consequently, the restriction $\Delta|_{e^{iM_{sa}}}$ admits a (unique) extension to a surjective real linear isometry from M onto N.

Corollary

The following statements are equivalent for any M and N unital JB*-algebras:

(a) M and N are isometrically isomorphic as (complex) Banach spaces;
(b) M and N are isometrically isomorphic as real Banach spaces;
(c) There exists a surjective isometry $\Delta : \mathcal{U}(M) \to \mathcal{U}(N)$.

Unitary Tingley's problem for JBW*-algebras

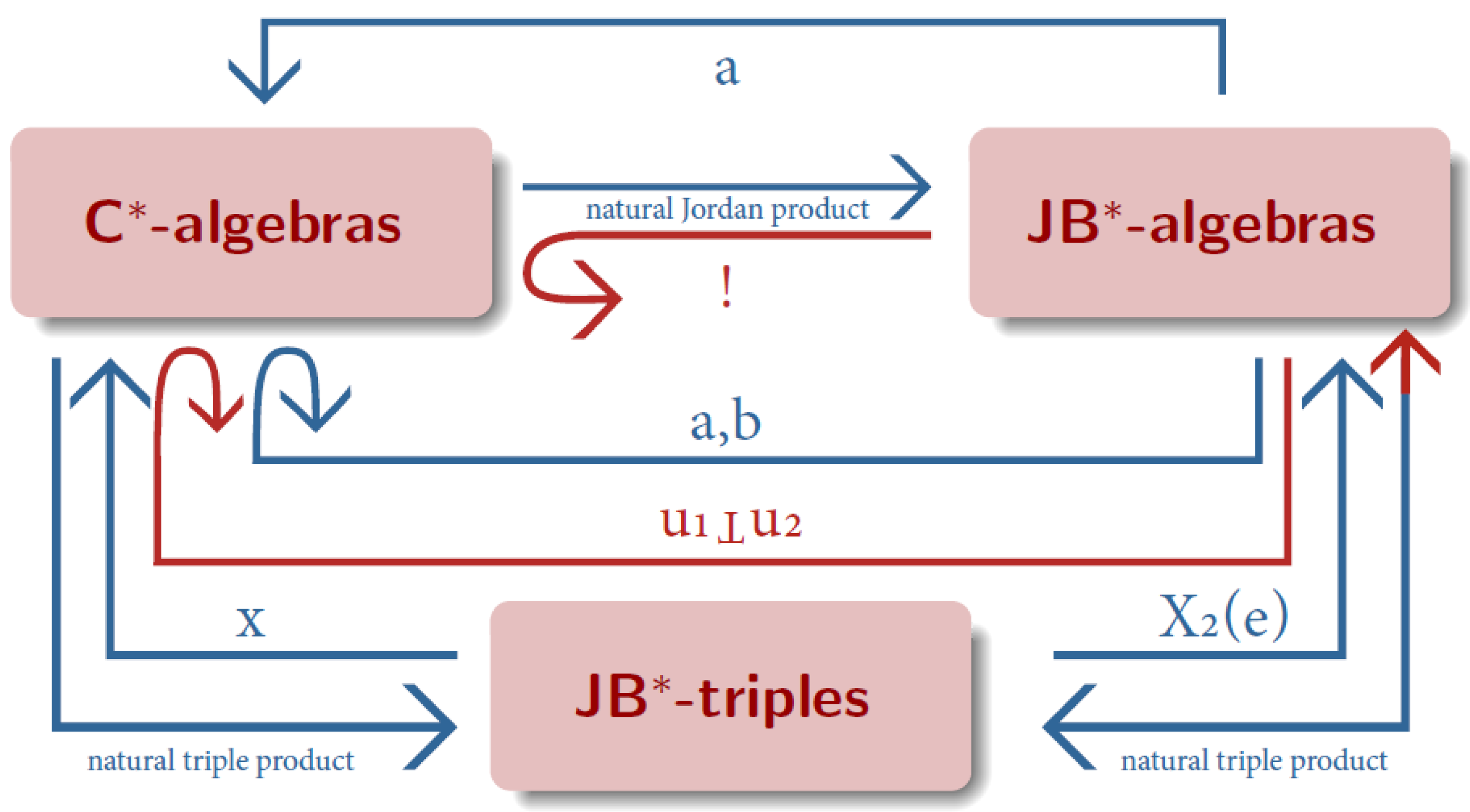
Let $\Delta : \mathcal{U}(M) \to \mathcal{U}(N)$ be a surjective isometry, where M and N are two JBW*-algebras. Then there exist a unitary ω in N, a central projection $p \in N$, and a Jordan *-isomorphism $\Phi : M \to N$ such that

$$\Delta(u) = U_{\omega^*}\left(p \circ \Phi(u)\right) + U_{\omega^*}\left((\mathbf{1}_N - p) \circ \Phi(u)^*\right), \quad \forall u \in \mathcal{U}(M).$$

Consequently, Δ admits a (unique) extension to a surjective real linear isometry from M onto N.

Jordan-Stone's one-parameter theorem

Let M be a unital JB*-algebra. Suppose $\{u(t) : t \in \mathbb{R}\}$ is a family in $\mathcal{U}(M)$ satisfying $u(0) = \mathbf{1}$, and $U_{u(t)}(u(s)) = u(2t + s)$, for all $t, s \in \mathbb{R}$. We also assume that the mapping $t \mapsto u(t)$ is continuous. Then there exists $h \in M_{sa}$ such that $u(t) = e^{ith}$ for all $t \in \mathbb{R}$.



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